SARDAR PATEL UNIVERSITY

Vallabh Vidyanagar-388120

B.Sc. (Semester - 3)

Subject: Physics

Course: US03CPHY21

Optics

(Four Credit Course –4 Hours per week)

(Effective from June-2019)

UNIT- I Geometrical Optics

Lens Systems: Introduction to lenses, Equivalent focal length of two thin lenses, Focal length of the equivalent lens, Distance of equivalent lens from L_2 and L_1, Powers, Cardinal points, Principal points and Principal planes, Focal points and Focal planes, Nodal points and Nodal planes, Construction of image using cardinal points, Newton's formula, Cardinal points of a coaxial system of two thin lenses- Object at infinity

Lens Aberrations: Introduction, Types of aberration, Spherical aberration, Reducing spherical aberration, Coma, Astigmatism, Curvature of field, Distortion, Chromatic aberration, Chromatic aberration in a lens- Object at infinity and object at finite distance

Eyepieces: Introduction to objective and eyepiece, Huygens eyepiece, Cardinal points of Huygens eyepiece, Ramsden eyepiece, Cardinal points of Ramsden eyepiece, Comparison of Ramsden and Huygens eyepieces

UNIT - II Interference and Diffraction

Interference: Introduction, Techniques for obtaining interference, Fresnel's biprism, Experimental arrangement, Determination of wavelength of light, Interference fringes with white light, Lateral displacement of fringes, Lloyd's single mirror, Determination of wavelength, Newton's ring, Condition for bright and dark rings, Circular fringes, Radii of dark fringes, Dark central spot, Determination of wavelength of light, Concept of multiple beam interference, Febry-Perot interferometer and Etalon, Formation of fringes, Determination of wavelength, Measurement of difference in wavelength, Lummer and Gehrcke plate

Diffraction: Introduction, Distinction between interference and diffraction, Fresnel and Fraunhoffer types of diffraction, Diffraction pattern due to a narrow slit, Diffraction due to a narrow wire, Fraunhoffer diffraction at a circular aperture, Fraunhoffer diffraction at double slit- Interference and diffraction maxima and minima

UNIT - III Polarization

Introduction, Polarized light, Production of linearly polarized light, Polarization by reflection, Polarization of refraction- pile of plates, Polarization by scattering, Polarization by selective absorption, Polarization by double refraction, Polarizer and analyzer, Construction and working of Nicol prism, Polaroid sheets, Effect of polarizer on natural light, Effect of analyzer on plane polarized light- Malus' law, Anisotropic crystals, Calcite crystal, Optic axis, Principle section, Double refraction, Huygens' explanation of double refraction, o-Ray and e-Ray, Positive crystals and negative crystals, Superposition of waves linearly polarized at right angles, Retarders or Wave plates, Quarter wave plate, Half wave plate, Production and detection of elliptically polarized light, Production and detection of circularly polarized light, Analysis of polarized light, Babinet compensator- construction and production of polarized light, Specific rotation, Laurent's half shade polarimeter, LCDs

UNIT- IV Fibre Optics

Introduction, Optical fibre, Necessity of cladding, Optical fibre system, Optical fibre cable, Total internal reflection, Propagation of light through an optical fibre, Critical angle of propagation, Acceptance angle, Fractional refractive index change, Numerical aperture, Modes of propagation, Classification of optical fibres, Single mode step index fibre, Multimode step index fibre, Graded index fibre, Materials, All glass fibres, All plastic fibres, PCS fibres, Bandwidth, Characteristics of the fibers, Applications, Illumination and image transmission, Optical communications, Medical applications, Military applications, Fibre optic communication system, Merits and demerits of optical fibers

Text Book:

1. A Textbook of Optics
   Subrahmanyam, Brij lal and Avadhnulu
   S Chand Publication (24th Revised addition 2010)

Reference Books:

1. Optics
   Ajoy Ghatak,
   McGraw-Hill Publishing Co. Ltd.
2. Textbook of light
   D N Vasudev
   Atma Ram and Sons, New Delhi
3. Fundamental of Optics
   F A Jenkine and H E White
   Tata McGraw Hill Book Co. Ltd.

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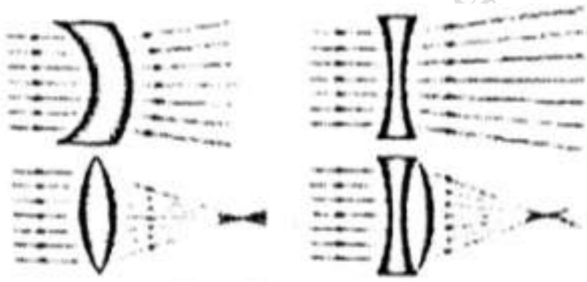
UNIT- I GEOMETRICAL OPTICS

[Lens systems (Chapter-4)**]:**

Introduction:

A lens is an image-forming device. It forms an image by refraction of light at its two bounding surfaces. In general, a lens is made of glass and is bounded by two regular curved surfaces; or by one spherical surface and a plane surface. Spherical surfaces are easy to make. Therefore, most lenses are made of spherical surfaces and have a wide range of curvatures. Other transparent materials such as quartz, fused silica and plastics are also used in making lenses. A single lens with two refracting surfaces is a simple lens. We study here the behaviour of a simple lens, with a view to gain familiarity with lens systems. We use the ray concept to understand the behaviour of light passing through lenses and derive the relationship between focal length of the lens, object distance and image distance.



Different types of lenses

LENSES:

Lenses are mainly of two types- convex lens and concave lens. A convex lens is thicker at the center than at the edges while a concave lens is thinner at the center than at the edges. A convex lens is a converging lens since a parallel beam of light, after refraction, converges to a point, F. A concave lens is called a diverging lens since rays coming parallel to the principal axis, after refraction, diverge out and seem to come from a point, F. Within these two categories there is a variety of simple lenses; some of the standard forms are shown in Fig. 4.1.

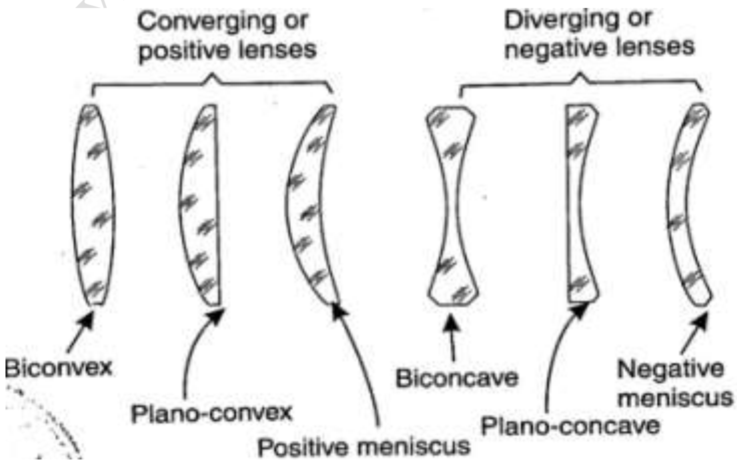


Fig - 4.1 Different types of lenses

Different types of lenses:

Converging or positive lenses:

Biconvex,
Plano-convex,
Positive meniscus,

Diverging or negative lenses:

Biconcave,
Plano-concave,
Negative meniscus

POWER:

The power of a lens is the measure of its ability to produce convergence of a parallel beam of light. A convex Iens of large focal length produces a small converging effect and a convex lens of small focal length produces a large converging effect. Due to this reason, the power of a convex lens is taken as positive and a convex lens of small focal length has high power. On the other hand, a concave lens produces divergence. Therefore, its power is taken as negative.

The unit in which the power of a lens is measured is called a diopter (D). A convex lens of focal length one meter has a power=+1 diopter and a convex lens of focal length 2 m has a power +0.5 diopter.

Mathematically,

$$\text{Power} = \frac{1}{\text{Focal length in meters}} \qquad \text{.........1}$$

The concept power is useful because it allows us to work out the effective focal length of a combination of lenses very easily. The power of a pair of lenses, of focal lengths f_1 and f_2 placed in contact is simply the sum of their individual powers

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{...............2}$$

$$\therefore P = P_1 + P_2 \qquad \text{...............3}$$

Where P_1 and P_2 are the powers of the two lenses and P is the equivalent power.

Equivalent focal length of two thin lenses:

When two thin lenses are arranged coaxially, the image formed by the first lens system becomes the object for a second lens system and the two systems act as a single optical system forming the final image from the original object.

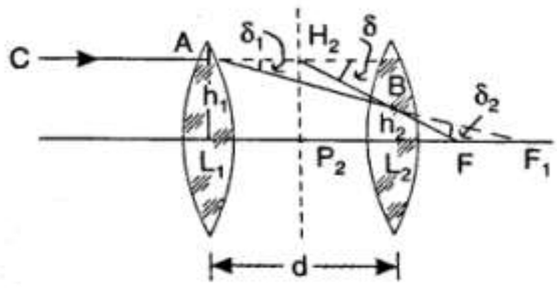


Fig. 4.14

Let us consider a simple optical system that consists of two thin lenses L_1 and L_2 placed on a common axis and separated by a distance d, as shown in Fig. 4.14. The lenses are separated by a distance and have focal lengths f_1 and f_2. We are interested to know how the combination works. We find that two lenses, separated by a finite distance, can be replaced by a single thin lens called an equivalent lens. The equivalent lens, when placed at a suitable fixed point, will produce an image of the same size as that produced by the combination of the two lenses. The focal length of equivalent lens is called equivalent focal length. We now derive an expression for the equivalent focal length, f, of the combination of two lenses.

Let a ray CA of monochromatic light parallel to the principal axis be incident on the first lens L_1 at a height h_1 above the axis. The ray CA is deviated through an angle δ_1 by the lens L_1. The incident ray CA, after refraction, is directed toward F_1, which is the second principal focus of lens L_1. Then the deviation produced by the first lens is given by

$$\delta_1 = \frac{h_1}{f_1} \qquad \ldots\ldots\ldots\ldots\ldots 4$$

The emergent ray, AB, from the first lens is incident on the lens L_2 at a height h_2. On refraction the ray is deviated through an angle δ_2, by the lens L_2 and meets the principal axis at F. Since the incident ray CA is parallel to the principal axis and after refraction through the optical system meets the axis at F. F must be the second principal focus of the combined lens system. The deviation produced by the second lens is given by

$$\delta_2 = \frac{h_2}{f_2} \qquad \ldots\ldots\ldots\ldots\ldots 5$$

If the incident ray CA is extended forward and the final emergent ray BF backward, they meet at a point H_2. It is clear that a single thin lens placed at P_2, will produce the same deviation as that produced by the two lenses put together. The lens of focal length P_2F placed at P_2 is termed as the equivalent lens, which can replace the two lenses L_1 and L_2. The deviation produced by the equivalent lens is

$$\delta = \frac{h_1}{f} \qquad \ldots\ldots\ldots 6$$

FOCAL LENGTH OF THE EQUIVALENT LENS:

Deviation produced by the first lens L is $\delta_1 = \frac{h_1}{f_1}$

Deviation produced by the second lens L is $\delta_2 = \frac{h_2}{f_2}$

But $\delta = \delta_1 + \delta_2$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \qquad \ldots\ldots\ldots\ldots\ldots 7$$

The Δ^{les} AL_1F_1 and BL_2F_1 are similar.

Or

$$\frac{AL_1}{L_1F_1} = \frac{BL_2}{L_2F_1}$$

Or

$$\frac{h_1}{f_1} = \frac{h_2}{f_1 - d} \qquad \ldots\ldots\ldots\ldots\ldots 8$$

$$h_2 = \frac{h_1(f_1 - d)}{f_1} \qquad \ldots\ldots\ldots\ldots\ldots 9$$

Using equation-9 into equation-7, we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1(f_1 - d)}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{f_1 - d}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad \ldots\ldots\ldots\ldots\ldots 10$$

Therefore, the equivalent focal length is given by

$$f = \frac{f_1 f_2}{f_1 + f_1 - d} \qquad \ldots\ldots\ldots\ldots\ldots 11$$

or

$$f = \frac{f_1 f_2}{\Delta} \qquad \ldots\ldots\ldots\ldots\ldots 12$$

Where $\Delta = d - (f_1 + f_2)$ and is known as the optical interval between the two lenses. It is numerically equal to the distance between the second principal focus of the first lens and the first principal of the second lens.

From Fig- 4.14, it may be seen that the rays converge at F and appear to have come from a plane EP_2. Plane EP_2 is therefore, the position of a single lens, the effect of which is the same that of the two lenses, L_1 and L_2. The distance L_2F is the equivalent focal length. The equivalent focal length is independent of the direction from which the light enters the system. The behaviour of the system cannot be reproduced in all respects by the equivalent thin lens placed in a fixed position. It will have to be placed in the position H_2P_2 when light enters from left and in position H_1P_1 when light enters from the right.

Note:

1. Equation-11 shows that if the distance d between the two convex lenses exceeds the sum of their focal lengths $(f_1 + f_2)$, the system becomes divergent, because of negative focal length.
2. If the medium between the two convex lenses is other than air, then the Equation-11 for equivalent focal length would become

$$f = \frac{f_1 f_2}{f_1 + f_2 - d/\mu}$$

Where μ is the refractive index of the medium

Distance of equivalent lens from L_2:

Let us say the plane EP_2, is located at a distance of L_2P_2 from the second lens L_2. Now consider the similar Δ^{les} EP_2F and BL_2F.

$$\frac{EP_2}{BL_2} = \frac{P_2F}{L_2F}$$

From Fig. 4.15, it is seen that $P_2F = f$, $L_2F = f - L_2P_2$ and $EP_2 = AL = h_1$

$$\frac{h_1}{h_2} = \frac{f}{f_1 - L_2P_2} \qquad \ldots\ldots\ldots\ldots\ldots 13$$

Comparing equations-13 and 8, we obtain

$$\frac{f}{f_1 - L_2P_2} = \frac{f_1}{f_1 - d}$$

$$ff_1 - fd = ff_1 + (L_2P_2)f_1$$

Or

$$L_2P_2 = \frac{fd}{f_1} \qquad \ldots\ldots\ldots\ldots\ldots 14$$

Distance of equivalent lens from L_1:

The distance of equivalent lens from L is given by

$$L_1P_2 = (d - L_2P_2) = \left(d - \frac{fd}{f_1}\right) = d\left(1 - \frac{f}{f_1}\right)$$

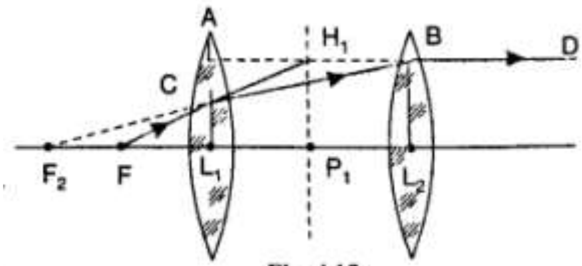


Fig. 4.15

If we consider a parallel beam of light to be incident from right and falls first on lens L, the equivalent lens moves to the left, as shown in Fig. 4.15.

The equivalent lens is now at a distance of L_1P_1 from the lens L_1 is given by

$$L_1P_1 = \frac{fd}{f_2} \qquad \ldots\ldots\ldots\ldots\ldots 15$$

These two positions of the equivalent lens are called the principal planes.

POWER:

When two thin lenses of focal lengths f_1 and f_2, are placed coaxial and separated by a distance d, the equivalent focal length is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

$$P = P_1 + P_2 - d(P_1 P_2) \qquad \ldots\ldots\ldots\ldots\ldots 16$$

[LENS SYSTEMS (Chapter-5)**]:**

Introduction to lens systems:

A lens is made of glass and is bounded by two regular curved surfaces; or one spherical surface and other plane surface. Lens is an image forming device. It forms an image by refraction of light at its two bounding surfaces. A single lens with two refracting surfaces is a simple lens.

As it is easy to make spherical surfaces, most of the lenses are made of spherical surfaces and have a wide range of curvatures. Single lens are rarely used for image formation, as they suffer from various defects. In optical instruments, such as microscopes, telescopes, cameras etc., a collection of lenses are used for forming images of objects. An optical system consists of number of lenses placed apart, and having a common principal axis. The image formed by such a coaxial optical system is good and almost free of aberrations.

Cardinal points of lens systems:

In 1841 Gauss showed that any number of coaxial lenses can be treated as a single unit, without the necessary of treating the single surfaces of lenses separately. The lens makers' formula can be applied to measure the distances from two hypothetical parallel planes. The points of intersection of these planes with the axis are called the principal points or Gauss points.

There are six points: (i) Two focal points, (ii) Two principal points and (iii) Two nodal points. These six points are known as Cardinal points of an optical system. The planes passing through these points and which are perpendicular to the principal axis are known as cardinal planes. Cardinal points and cardinal planes are intrinsic properties of a particular optical system and determine the image forming properties of the system. If these are known then one can find the image of any object without making a detailed study of the passage of the rays through the system.

Principal points and Principal planes:

Consider an optical system having its principal focal points F_1 and F_2. A ray OA traveling parallel to the principal axis and incident at A is brought to focus at F_2 in the image space of the optical system as shown in the given figure (3).

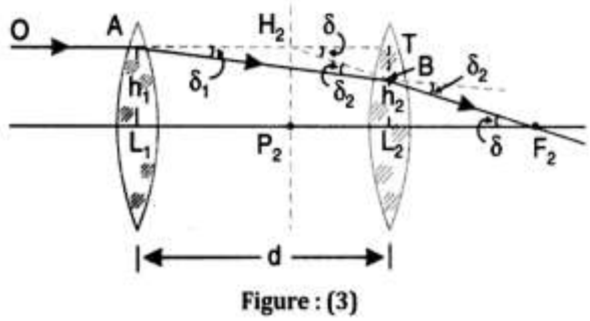


Figure : (3)

The actual ray is refracted at each surface of the optical system and follows the path $OABF_2$. If we extend the incident ray OA forward and the emergent ray BF_2 backward, they meet each other within the optical system at H_2. Now we can describe the refraction of the incident ray OA in terms of a single refraction at a plane passing through H_2. A plane drawn through the point H_2 and perpendicular to the axis may be considered as the surface at which refraction takes place. This plane is called the **principal plane of the optical system**. Thus, the four consecutive deviations of the light rays caused by the four surfaces of the optical system are equivalent to single refraction at H_2, taking place at the **principal plane.**

Thus now we can define the principal plane of an optical system as the loci where we assume refraction to occur without reference as to where the refraction actually occurs. H_2P_2 is the principal plane in the image space and is called the **second principal plane.** The point P_2 at which the second principal plane intersects the axis, is called the **second principal point.**

Now consider figure (4), one can locate the principal plane H_1P_1 and principal point P_1in the object space. Consider the ray F_1S passing through the first principal focus F1 such that after refraction it emerges along QW parallel to the axis at the same height as that of the ray OA (as in figure (3). The rays F_1S and QW when produced intersect at H_1. A plane perpendicular to the axis and passing through H_1 is called the **first principal plane.** The point of intersection, P1, of the first principal plane with the axis is called the **first principal point.**

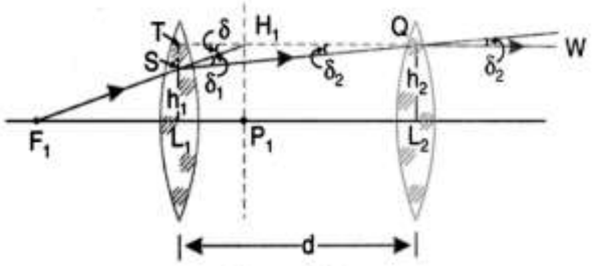


Figure : (4)

Consider figure (5) the two incident rays (F_1S & OA) are directed towards H1 and after refraction seem to come from H_2. Therefore, H_2 is the image of H1. Thus H1 and H2 are the conjugate points and the planes H_1P_1 and H_2P_2 are pair of **conjugate planes.** It has also been observed that $H_2P_2 = H_1P_1$.

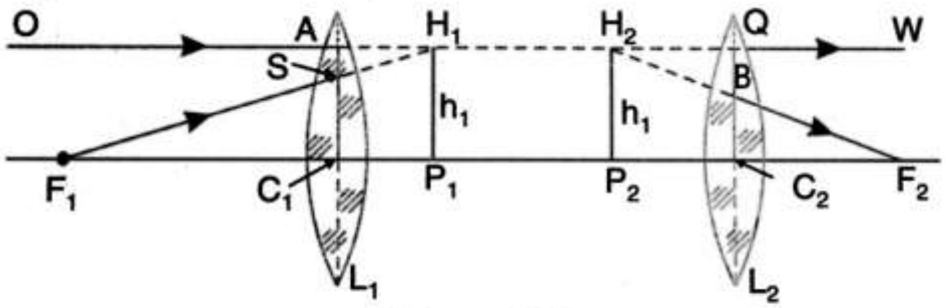


Figure : (5)

Thus lateral magnification of the planes is +1. Thus, the first and second principal planes are planes of unit magnification and are therefore called as unit planes and points P_1 and P_2 are called unit points. Note that principal planes are conceptual planes and do not have physical existence within the optical system.

- **Some remarkable features of principal planes:**
  1) Even a complex optical system has only **two** principal planes.
  2) Between H_1 and H_2 all rays are parallel to the principal axis.
  3) The location of the principal planes is characteristics of a given optical system/their positions do not change with the object and image distance used.
  4) The principal points H_1 and H_2 provide a set of references from which several system parameters are measured.
  5) The principal planes are conjugate to each other. An object in the first plane is imaged in the second principal plane with unit magnification. Any ray directed towards a point on the first principal plane emerges from the lens as if it originated at the corresponding point (at the same distance

above and below the axis) on the second principal plane. Hence, the name unit planes.

Focal points and focal planes:

When a parallel beam of light traveling from the left to the right (in object space) is incident on the optical system, the beam comes together at a point F_2, on the other side (in image space) of the optical system. The beam passes through the point F_2, wherever may be its path inside the system. The point F_2, is called the **second focal point** of the optical system.

A beam of light passing the point F_1 on the axis of the object side is rendered parallel to the axis after emergence through the optical system see figure (5), The point F_1 is called the **first focal point.**

Thus the focal points can be defined as

(i) **The first focal point:** The first focal point is a point on the principal axis of optical system such that a beam of light passing through it is rendered parallel to the principal axis after refraction through the optical system.

(ii) **The second focal point:** The second focal point is a point on the principal axis of the optical system such that a beam of light traveling parallel to the principal axis of the optical system, after refraction through the system, passes through it.

The planes passing through the principal focal points F_1 &F_2 and perpendicular to the axis are called first focal plane and second focal plane respectively. The main property of the focal planes is that the rays starting from a point in the focal plane in the object space corresponds to a set of conjugate parallel rays in the image space. Similarly, a set of parallel rays in the object space corresponds to a set of rays intersecting at a point in the focal plane in the image space.

The distance of the first focal point from the first principal point, i.e. F_1P_1 is called the first focal length, f_1 of the optical system and the distance of the second focal point from the second principal point, F_2P_2 is called the second focal length, f_2, f_1 and f_2 are also known as the focal lengths in the object space and image space respectively. When the medium is same on the two sides of the optical system $f_1=f_2$.

Nodal points and Nodal planes:

Nodal points are the points on the principal axis of the optical system where light rays, without refraction, intersect the optic axis. In a thin lens the nodal point is the centre of the lens. Light passing through the centre of a thin lens does not deviate. In an optical system the centre separates into two nodal points. The planes passing through the nodal points and perpendicular to the principal axis are called the nodal planes. Where as the principal planes are the planes where all refractions are assumed to occur, the nodal planes are planes

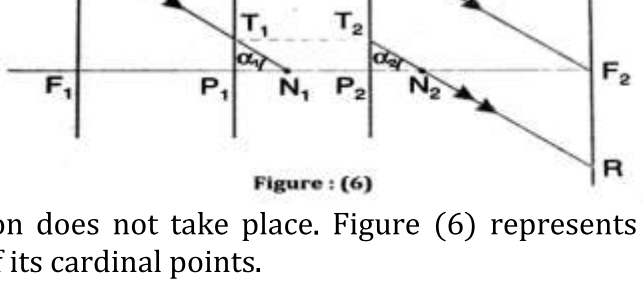


Figure : (6)

where refraction does not take place. Figure (6) represents an optical system with the help of its cardinal points.

A ray of light, AN_1, directed, towards one of the nodal points N_1, after refraction through the optical system, along N_1N_2, emerges out from the second nodal point N_2, in a direction, N_2R, parallel to the incident ray. The distances of the nodal points are measured from the focal points.

The nodal points are a pair of conjugate points on the axis having unit positive angular magnification

Let H_1P_1 and H_2P_2 be the first and second principal planes of an optical system. Let AF_1 and BF_2 be the first and second focal plane respectively. Consider a point a situated on the first focal plane. From A draw a ray AH_1 parallel to the axis. The conjugate ray will proceed from H_2, appoint in the second principal plane such that $H_2P_2 = H_1P_1$ and will pass through the second focus.

Take another ray AT_1 parallel to the emergent ray H_2F_2 and incident on the first principal plane at T_1. It will emerge out from T_2, a point on the second principal plane such that $T_2P_2 = T_1P_1$, and will proceed parallel to the ray H_2F_2. The point of intersection of the incident ray AT_1 and the conjugate emergent ray T_2R with the axis give the positions of the nodal points. It is clear that two points N_1 and N_2 are a pair of conjugate points and the incident ray AN_1 is parallel to the conjugate emergent ray T_2R.

Further $\tan\alpha_1 = \tan\alpha_2$

The ratio $\frac{\tan\,\alpha_2}{\tan\,\alpha_1} = \gamma$ represents the angular magnification.

$$\therefore\ \frac{\tan\,\alpha_2}{\tan\,\alpha_1} = 1 \qquad \text{.................12}$$

Therefore, the points N_1 and N_2 are a pair of conjugate points on the axis having unit positive angular magnification.

The distance between two nodal points is always equal to the distance between two principal points

Referring figure (6), we see that in the right angled Δ^{les} $T_1P_1N_1$ and $T_2P_2N_2$

$T_1P_1 = T_2P_2$

$\angle T_1P_1N_1 = \angle T_2P_2N_2 = \alpha$

Therefore, the two Δ^{les} are congruent.

$\therefore\ P_1N_1 = P_2N_2$

Adding N_1P_2 to both the sides, we get

$\therefore\ P_1N_1 + N_1P_2 = P_2N_2 + N_1P_2$

$$\therefore\ P_1P_2 = N_1N_2 \qquad \text{.....13}$$

Thus, the distance between the principal points N_1 and N_2 is equal to the distance between the principal points P_1 and P_2.

- **The nodal points N_1 and N_2 coincide with the principal points P_1 and P_2 respectively whenever the refractive indices on either side of the lens are the same.**

Now consider the two right angled Δ^{les} $T_1P_1N_1$ and $T_2P_2N_2$ in figure (6).

$AF_1 = H_2P_2$

$\angle AN_1F_1 = \angle H_2F_2P_2$

$\therefore$ The two Δ^{les} are congruent.

$F_1N_1 = P_2F_2$

But

$F_1N_1 = F_1P_1 + P_1N_1$

$\therefore F_1P_1 + P_1N_1 = P_2F_2$
$\therefore P_1N_1 = P_2F_2 - F_1P_1$

Also

$P_2F_2 = +f_2$ and $P_1F_1 = -f_1$

$\therefore P_1N_1 = P_2N_2 = (f_1 + f_2)$

As the medium is the same, say air, on both the side of the system

$f_2 = -f_1$

$\therefore P_1N_1 = P_2N_2 = 0$ ……….14

Thus, the principal points coincide with the nodal points when the optical system is situated in the same medium.

Construction of image using cardinal points:

We have seen the different types of cardinal points of a optical system. Having these ideas one can construct the image corresponding to any object placed on the principal axis of the system. It is not necessary to know the position and curvatures of the refracting surfaces or the nature of the intermediate media. Only knowledge of cardinal points and cardinal planes is sufficient.

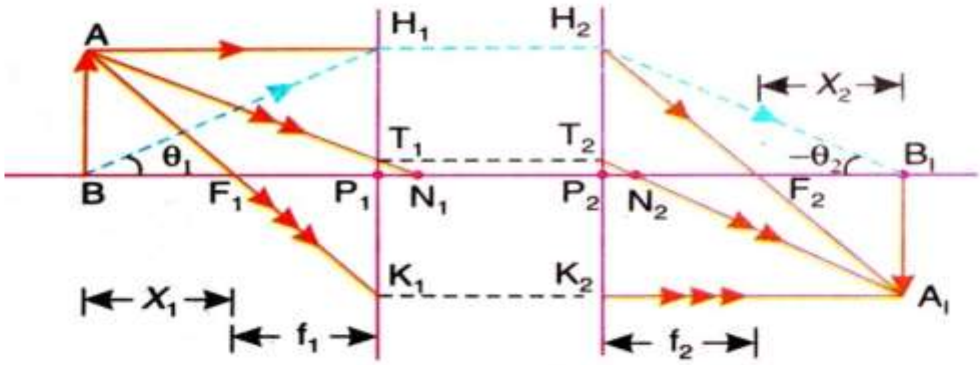


Figure : (7)

Suppose F_1, F_2 be the principal foci, P_1, P_2 the principal points and N_1, N_2 the nodal points of the optical system as shown in the given figure (7). AB is a linear object on the axis. Now in order to find the image of the point A we make the following construction.

i) A ray AH_1 is drawn parallel to the axis touching the first principal plane at H_1. The conjugate ray will proceed from H_2 a point on the second principal plane such that $H_2P_2 = H_1P_1$ and will pass through the second principal focus F_2.

ii) A second ray AF_1K_1 is drawn passing through the first principal focus F_1 and touching the first principal plane at K_1. Its conjugate ray will proceed from K_2 such that $K_2P_2 = K_1P_1$ and it will be parallel to the axis.

iii) A third ray $A_1T_1N_1$ is drawn which is directed towards the first nodal pointN1. This ray passes after refraction through N_2 in a direction parallel to AN_1.

The point of interaction of any of the above two refracted rays will give the image of A. Let it be A_1 from A_1, if a perpendicular is drawn on to the axis , it gives the image A_1B_1 of the object AB.

Newton's formula:

Consider the given figure (7), it is seen that $\Delta^{les}\ ABF_1$ and $F_1K_1P_1$ are similar

$$\frac{K_1P_1}{AB} = \frac{P_1F_1}{BF_1}$$

But $K_1P_1 = A_1B_1$

$$\therefore\ \frac{A_1B_1}{AB} = \frac{f_1}{x_1} \quad \text{.........15}$$

Further, $\Delta^{les}\ A_1B_1F_1$ and $H_2P_2F_2$ are similar

$$\frac{A_1B_1}{H_2P_2} = \frac{B_1F_2}{P_2F_2}$$

But $H_2P_2 = AB$

$$\frac{A_1B_1}{AB} = \frac{x_2}{f_2} \quad \text{......16}$$

From equation (15) and (16) we get

$$\therefore\ \frac{h_2}{h_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2} \quad \text{............17}$$

or

$$x_1x_2 = f_1f_2 \quad \text{......18}$$

This is the **Newton's formula**. In the foregoing discussion, the distances of the image and the object have been measured from their respective foci. But it is sometimes convenient to measure the conjugate distances from the principal points.

Combination of two thin lenses:

The cardinal points of a coaxial optical system is determined by assuming first that the object at infinity and then the case o object located at on the principal axis at a certain distance from the system. One has to see that the computations would yield identical results in both the cases.

- **Object at infinity:** Here we consider an object located at infinity, see given figure.

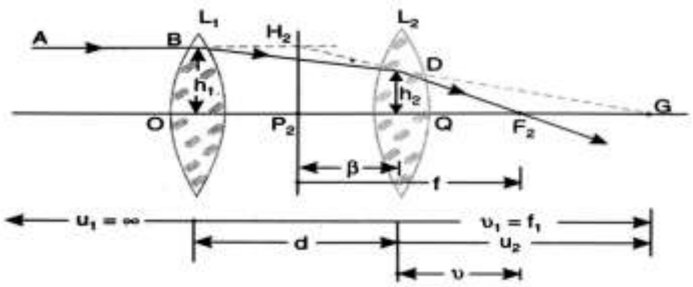


Figure : (8)

AB is a ray of light coming from an object situated at a very large distance, such that $u_1 = \infty$. The lens L_1, if alone, would form an image at G. However, because of the presence of the second lens L_2. G becomes the virtual object for L_2. The ray BD, instead of going along BDG, refracts along the path DF_2. When the ray AB is produced forward and the ray DF_2 backward, they intersect at H_1. The plane $H_1\ P_1$ normal to the axis may considered as the plane at which the refraction occurred and the plane is called **principal plane.**

Focal length of the system:

Now we can write the expression for the refraction taking place at the surface of first lens as follows:

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1} \qquad \therefore \frac{1}{OG}-\frac{1}{u_1}=\frac{1}{f_1}$$

As $u_1 = \infty$, we obtain OG $= f_1$

The equation for the refraction at the second lens may be written as

$$\therefore \frac{1}{v}-\frac{1}{u_2}=\frac{1}{f_2} \qquad \therefore \frac{1}{QF_2}-\frac{1}{QG}=\frac{1}{f_2}$$

or

$$\therefore \frac{1}{QF_2}=\frac{1}{f_2}+\frac{1}{f_1-d} \qquad \therefore \frac{1}{QF_2}=\frac{f_1 f_2 +-d}{f_1\left(f_2-d\right)} \qquad \text{.........19}$$

The Δ^{les} BOG and DQG are similar and also the Δ^{les} CP_1F_2 and DQF_2 are similar

$$\therefore \frac{BO}{OG}=\frac{DQ}{-QG} \qquad \therefore \frac{h_1}{f_1}=\frac{h_2}{-(f_1-d)} \qquad 20$$

$$\therefore \frac{CP_1}{P_1F_2}=\frac{DQ}{-QF_2} \qquad \therefore \frac{h_1}{f}=\frac{h_2}{QF_2} \qquad 21$$

From equation (19), (20) and (21) we get

$$\frac{h_1}{h_2}=\frac{f_1}{-(f_1-d)}=\frac{f(f_1+f_2+-d)}{f_2(f_1-d)}$$

or

$$\frac{1}{f}=\frac{f_1+f_2+-d}{f_1 f_2} \qquad 22$$

$$\therefore \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \qquad 23$$

The location of focal point F_2 is determined by QF_2, which is known from the equation, the position of the principal plane P_1 is specified by the value of f calculated from equation (23).

Cardinal points

(i) Second principal point: Let us say the second principal plane H_2P_2 is located at a distance of $L_2P_2 = \beta$ from the second lens L_2. According to sign convention β would be negative as it is measured toward the left of lens.
$\therefore QF_2 = f-(-\beta) = f+\beta$
We can determine β using the equation for f into the above relation. Thus,

$$f+\beta=\frac{f_2(f_1-d)}{f_1+f_2-d}$$

$$\beta=-f+\frac{f_2(f_1-d)}{f_1+f_2-d}=\frac{-f_1 f_2}{\Delta}+\frac{f_2 f_1-f_2 d}{\Delta}$$

Where $\Delta = f_1+f_2-d$

$$\therefore\ \beta=-f_2\frac{d}{\Delta} \qquad \text{... ... ... 24}$$

or

$$\beta = -\frac{f_2 d}{f_1 + f_2 - d} \quad \text{.....25}$$

But from eq. (22), we have $f_1 + f_2 - d = \frac{f_1 f_2}{f}$

$$\beta = -\frac{fd}{f_1} \quad \text{........26}$$

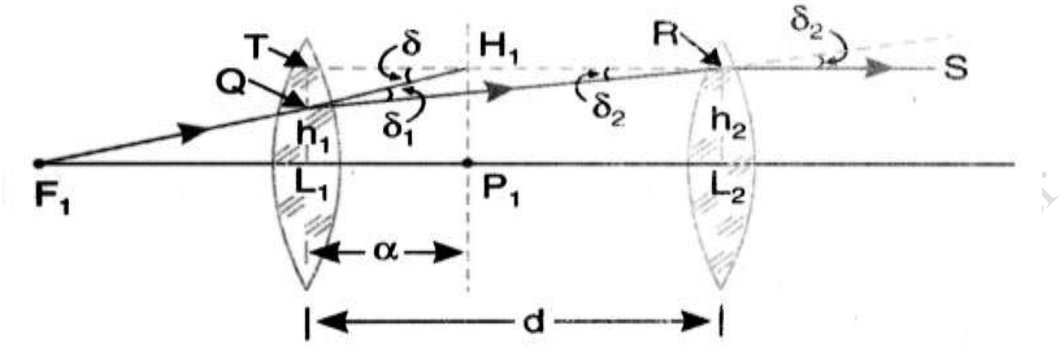


Figure : (9)

(ii) First principal point :

By considering a ray of light parallel to the axis and incident on the second lens L_2 from the right side, we can show that the distance of first principal plane. $L_1P_1 = \alpha$, from the first lens L_1 is given by

$$\alpha = f_1 \frac{d}{\Delta} \quad \text{.........27}$$

Also,

$$\alpha = +\frac{fd}{f_2} \quad \text{..........28}$$

This is the same as the result.

(iii) Second focal point :

Referring to figure (8), the distance of the second focal point F_2 from the second lens L_2 is given by

$$L_2F_2 = P_2F_2 - P_2L_2$$

$$= f - (-L_2P_2) = f + \beta$$

$$= f + \left(-\frac{fd}{f_1}\right)$$

$$\therefore \; L_2F_2 = f\left(1 - \frac{d}{f_1}\right) \quad \text{.................29}$$

(iv) First focal point :

The distance of the first focal point F_1 from the first lens L_1 is given by

$$L_1F_1 = P_1F_1 - P_1L_1$$

$$= -f - (-L_1P_1) = -f + \alpha$$

$$= -f + \left(\frac{fd}{f_2}\right)$$

$$\therefore\ L_1F_1 = -f\left(1 - \frac{d}{f_2}\right) \qquad \text{.........30}$$

(v) and (vi) nodal points :
As the optical system is considered to be located in air, P_1 and P_2 are also the positions of nodal points N_1 and N_2 respectively.

Lens Aberrations:

One of the basic problems of lenses is the imperfect quality of the image. In actual practice the objects are bigger and a lens is required to produce a bright and magnified image. We are therefore, required to take into consideration the wide angle rays from the centre of the object and also the upper and lower parts of the object and falling near the top and bottom of the lens.

These rays are known as peripheral or marginal rays. In general, a) peripheral rays of light do not meet at a single point after refraction through the lens. b) Secondly, the refractive index and hence the focal length of a lens is different for different wave lengths of the light.

For a given lens, the refractive index for violet light is more than that for red light. Thus if the light coming from an object point is not monochromatic, the lens forms a number of coloured images. These images even though formed by paraxial rays, are at different positions and are different sizes.

Types of monochromatic aberration:

The deviation of real images from the ideal images, in respect of the actual size, shape and position are called aberrations. One can also say that an aberration is any failure of a mirror or a lens to behave precisely according to the simple formulae we have derived. Aberrations are only due to inherent shortcomings of a lens and not caused by faulty construction of the lens, such as irregularities in its surfaces. They are inevitable consequences of the laws of refraction at spherical surfaces.

Aberrations are divided broadly into two categories – monochromatic aberrations and chromatic aberrations. The defects due to wide-angle incidence and peripheral incidence which occur even with monochromatic light are called monochromatic aberrations. Aberrations occur due to dispersion of light are called chromatic aberrations. Chromatic aberration occurs with light that contains at least two wavelengths. Monochromatic aberrations are again divided in to five types and they are

i. Spherical aberrations
ii. Coma
iii. Astigmatism
iv. Curvature of field
v. Distortion

The deviations from the actual size, shape and position of an image as calculated from the earlier simple equations are called the aberrations produced by lens. The aberrations produced by the variation of refractive index with wave length of light are called chromatic aberrations. The other aberrations are caused even if monochromatic light is used and they are called monochromatic aberrations. Lens aberrations are just the consequence of the refraction laws at the spherical surfaces and not due to defective construction of a lens such as the surfaces and not due to defective construction of a lens such as the surfaces being not spherical etc.

Spherical aberration:

A lens may be regarded as made up of a large number of prisms, of increasing angles from the centre to outward in case of a convex lens and of decreasing angles in case of a concave lens. A ray of light falling on a prism of larger angle is deviated more towards the base of the prism than that falling on a prism of small angle. Therefore, peripheral light rays passing through a lens farther away from the axis are refracted more and come to focus closer to the lens. Paraxial rays passing through the lens close to the axis are refracted less and come to focus farther from the lens. Therefore, rays passing through different zones of a lens surface come to different foci. **An image formed by paraxial rays will be surrounded by a diffuse halo formed by peripheral rays and consequently the image is blurred. This phenomenon is known as Spherical aberration.**

Consider the given figure (10), which shows the presence of spherical aberration in the image formed by a single lens. Here O is a point object on the axis of the lens and I_p and I_m are the images formed by the paraxial and marginal rays respectively. One can see from the figure very clearly that paraxial ray of light from the image at a longer distance from the lens than the marginal rays.

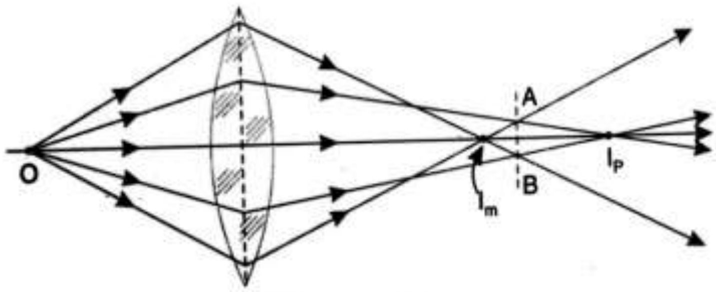


Figure : (10)

The image is not sharp at any point on the axis. However, if the screen is placed perpendicular to the axis at AB, the image appears to be a circular patch of diameter AB. At positions on the two sides of AB, the image patch has a large diameter. The patch of diameter AB is called the **circle of least con fusion**, which corresponds to the position of the best image.

The distance I_m I_p measures the **longitudinal spherical aberration**. The radius of the circle of least confusion measures the **lateral spherical aberration.**

When the aperture of the lens is relatively large compared to the focal length of the lens, the cones of the rays of light refracted through the different zones of the lens surface are not brought to focus at the same point I_m and the axial rays come to a focus at farther point I_p. Thus, for an object point O on the axis, the image extends over the length I_m I_p. This effect is called **spherical aberration** and arises due to the fact that different annular zone have different focal lengths. Figure (11) shows the spherical aberration produced by a concave lens.

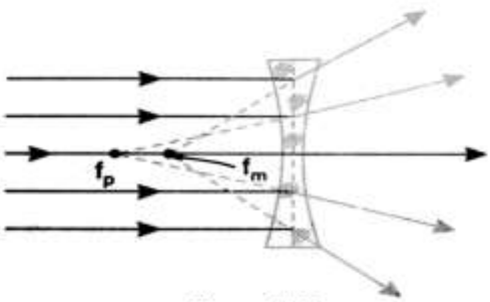


Figure : (11)

The spherical aberration produced by a lens depends on the distance of the object point and varies approximately as the square of the distance of the object ray above the axis of the lens. The **spherical aberration** produced by a **convex lens** is **positive** and that produced by a **concave lens** is **negative.**

Reducing spherical aberration:

Spherical aberration produced by lenses is minimized or eliminated by the following ways and means

i) Spherical aberration can be minimized by using stops, which reduces the effective lens aperture. The stop used can be such as to permit either the axial rays of light or the marginal rays of light. However, as the amount of light passing through the lens is reduced, correspondingly the image appears less bright.

ii) The longitudinal spherical aberration produced by a thin lens for a parallel incident beam is given by

$$x = \frac{\rho_2}{f_2}[\frac{k^2\mu^2 + (\mu + 2\mu^2 + 2\mu^3) + \mu^3 - 2\mu^2 - 2}{2\mu\ (\mu\ -1)^2(1-k)^2} \quad \text{.........31}$$

Where, x is the longitudinal spherical aberration. P is the radius of the lens aperture and f2 is the second principal focal length.

$$k = \frac{R_1}{R_2}$$

Where, R1 and R2 are the radii of curvature.

For given values of μ, f_2 and ρ, the conditions for minimum spherical aberration is $\frac{dx}{dk} = 0$

Differentiating equation (31) and equating the result to zero, we get

$$k = \frac{R_1}{R_2} = \frac{\mu(2\mu - 1) - 4}{\mu(2\mu + 1)} \quad \text{........32}$$

From equation (32), for a lens whose material has refractive index μ=1.5, k= - 1/6. Thus, the lens, which produces minimum spherical aberration, is biconcave and the radius of curvature of the surface facing the incident light is one-sixth the radius of curvature of the other face. In general, the more curved surface of the lens should face the incident or emergent beam of light whichever is more parallel to the axis. Lens whose $\frac{R_1}{R_2} = -\frac{1}{6}$ is called a **cross lens.** The process in which the shape of the lens is changed without changing the focal length of the lens is called bending of the lens for minimum spherical aberration. A crossed lens is shown in figure (12).

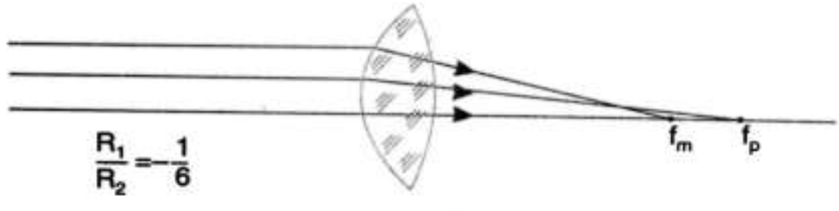


Figure : (12)

iii) Plano-convex lenses are used in optical instruments so as to reduce the spherical aberration. When the curved surface of the lens faces the incident or emergent light whichever is more parallel to the axis, the spherical aberration is minimum. The spherical aberration in crossed lens $\frac{R_1}{R_2} = -\frac{1}{6}$ is only 8% less than that of a Plano-convex lens having the same focal length and radius of the lens aperture. This is the reason why Plano-convex lenses are generally used in place of crossed lenses without increasing the spherical aberration appreciably. Figure (13) represents the variation of longitudinal spherical aberration with the radius of the lens aperture for lenses of the same focal length and refractive index.

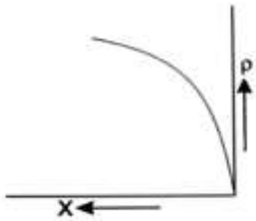


Figure : (13)

The spherical aberration will, however, be very large if the plane surface faces the incident light. The spherical aberration is a result of larger deviation of the marginal rays than the paraxial rays. If the deviation of the marginal rays of light is made minimum, the focus f_m for a parallel incident beam will shift towards f_p, the focus for the paraxial rays of light and the spherical aberration will be minimum.

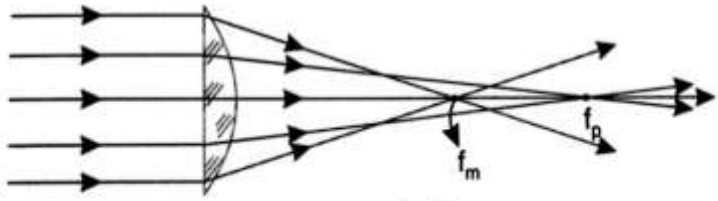


Figure : (14)

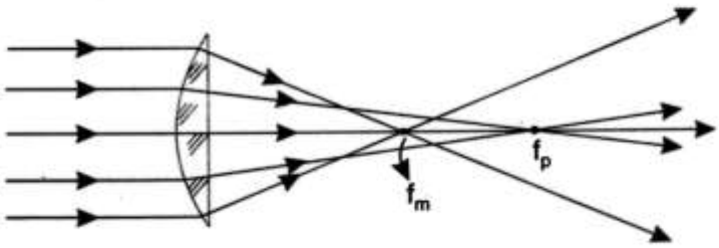


Figure : (15)

As the deviation is a minimum in a prism, when the angles of incidence and emergence are equal, similarly in a lens also, spherical aberration can be minimized if the total deviation produced by a lens is equally shared by the two surfaces.

In Plano-convex lens, when the plane surface faces the parallel beam of light, the deviation is produced only at the curved surface and hence the longitudinal spherical aberration (see figure (14)) is more than when the curved surface faces the incident light, as shown in figure (15).

The spherical aberration produced by a by a single lens can be minimized by choosing proper radii of curvature. The shape factor q of the lens is given by $q = \dfrac{R_1 + R_2}{R_2 - R_1}$

The spherical aberration for a double convex lens (shape factor q=0.5) is a minimum when the surface of smaller radius of curvature faces the incident parallel light. The spherical aberration for a Plano-convex lens (shape factor q=+1.0) when the curved surface faces the incident light is only slightly more than the double convex lens. Hence, Plano-convex lenses are preferred.

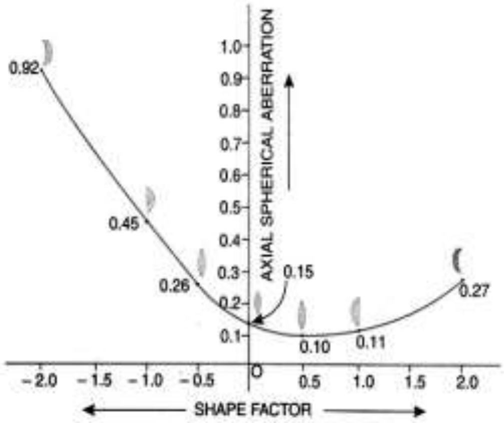


Figure : (16)

iv) Spherical aberration can also be made minimum by using two Plano-convex lenses separated by a distance equal to the difference in their focal length. In this arrangement, the two lenses equally share the total deviation and the spherical aberration is minimum. In figure (17), two Plano-convex lenses of focal length f1 and f2 are separated by a distance d.

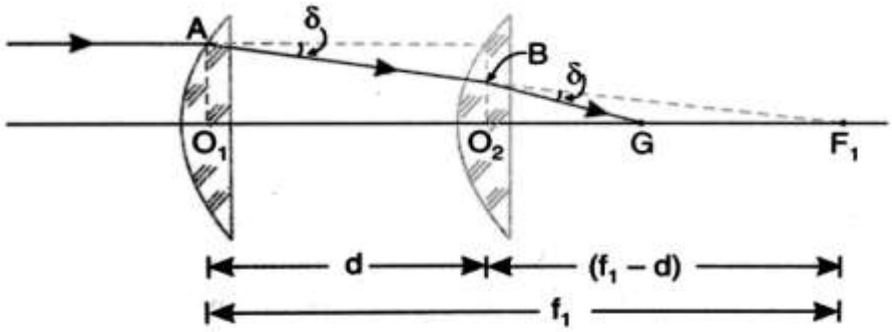


Figure : (17)

Let δ be the angle of deviation produced by each lens see figure (17).

$\angle BF_1G = \delta$, $\angle F_1BG = \delta$

And from triangle BGF_1, $BG = GF_1$ or $O_2G = GF_1$ (approximately)

$O_2G = \frac{1}{2}(O_2F_1) = \frac{1}{2}(f_1 - d)$

For the second lens F_1 is the virtual object and G is the real image. Substituting these values of object and image distances in formula

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2}$

$\frac{1}{O_2G} - \frac{1}{O_2f_1} = \frac{1}{f_2}$

$\frac{2}{(f_1 - d)} - \frac{1}{(f_1 - d)} = \frac{1}{f_2}$

$\frac{1}{(f_1 - d)} = \frac{1}{f_2}$

$f_2 = f_1 - d$ or $d = f_1 - f_2$ ………………..33

Thus, the condition for minimum spherical aberration is that the distance between the two lenses is equal to the difference in their focal lengths

v) Spherical aberration for a convex lens is positive and that for a concave lens is negative. By a suitable combination of convex and concave lenses spherical aberration can be made minimum.

vi) Spherical aberration may be minimized by using axial –GRIN lenses.

Coma:

The effect of rays from an object point not situated on the axis of the lens results in an aberration called Coma. Chromatic aberration is similar to spherical aberration in that both are due to the failure of the lens to bring all rays from a point object to focus at the same point. Spherical aberration refers to object points situated on the axis whereas Chromatic aberration refers to object points situated off the axis.

In the case of Spherical aberration, the image is a circle of varying diameter along the axis and in the case of Chromatic aberration the image is comet shaped and hence the name is coma.

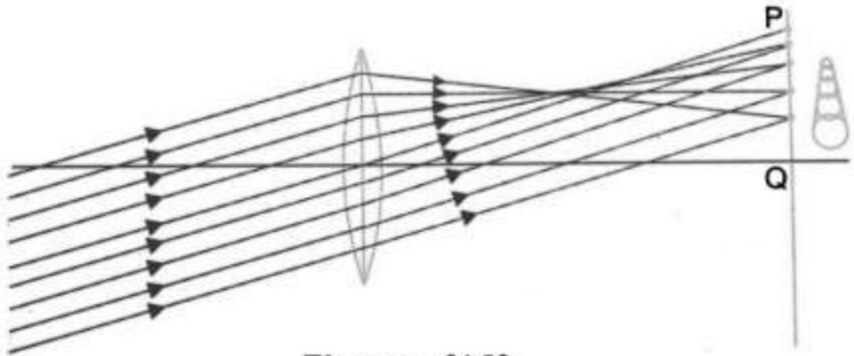


Figure : (18)

The given figure illustrates the effect of coma. The resultant image of a distant point off the axis is shown on the right side of the figure (18). The rays of light in the tangential plane are represented in the figure.

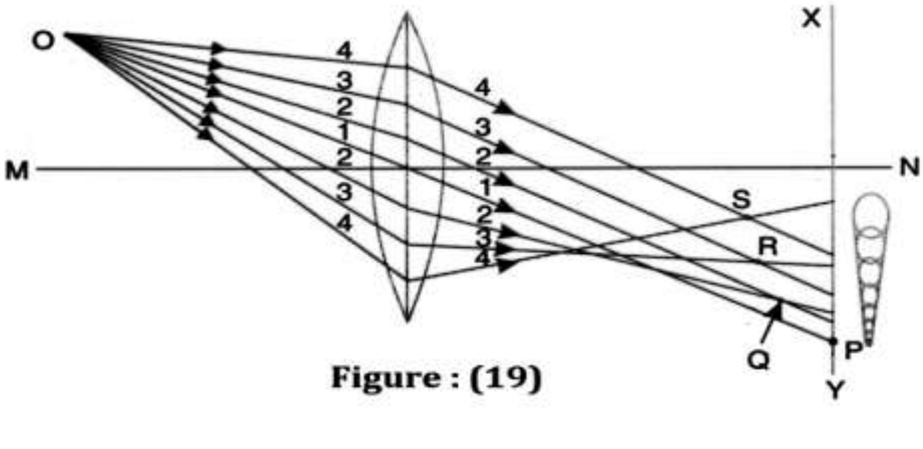


Figure : (19)

The figure (19) illustrates the presence of coma in the image due to a point object, o, situated off the axis of the lens. Rays of light getting refracted through the centre of the lens(ray 1) meet the screen XY at the point P. rays 2.2, 3.3, etc getting refracted through the outer zones of the lens come to focus at points Q,R,S, etc. , nearer the lens and overlapping circular patches of gradually increasing diameter are formed on the screen. The resultant image of the point is comet-shaped as shown on the right side of the respective figure.

Let 1, 2, 3 etc., be the various zones of the lens See the figure (20a). Rays of light getting refracted through these different zones give rise to circular patches of light 1', 2', 3', etc. The screen is placed perpendicular to the axis of the lens and at the position where the central rays come to focus. Figure (20b). Like spherical aberration, Chromatic aberration produced by a single lens can also be corrected by properly choosing the radii of curvature of the lens surface. Coma can be altogether eliminated for a given pair of object and image points where as spherical aberration cannot be completely corrected. Further, a lens corrected for coma will not be free from spherical aberration and the one corrected for spherical aberration will not be free from coma. Use of a stop or a diaphragm at the proper position eliminates coma.

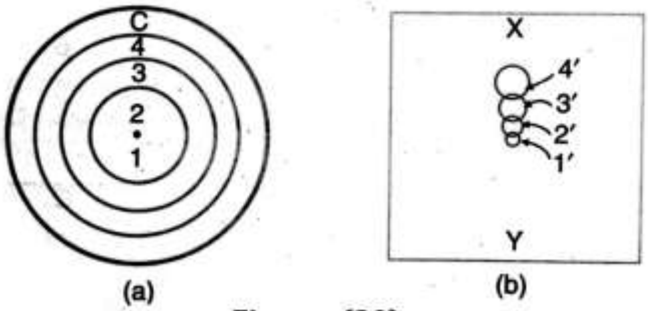


Figure : (20)

Coma is the result of varying magnification for rays refracted through different zones of the lens. For example, in figure (19), rays of light getting refracted through the outer zones come to focus at points nearer the lens. Hence the magnification of the image due to the outer zones is larger than the inner zones and in this case coma is said to be positive, on the other hand if the magnification produced in an image due to the other zones is similar, coma is said to be negative.

According to Abbe, a German optician, coma can be eliminated if a lens satisfies the Abbe's sine condition viz.

$$\mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2 \qquad \text{........34}$$

Where, μ_1, y_1 & θ_1 refer to the refractive index, height of the object above the axis, and the stop angle of the incident ray of light respectively. Similarly μ_2, y_2 & θ_2 refer to the corresponding quantities in the image space. The magnification of the image is given by $\frac{y_2}{y_1}$

$$\frac{y_2}{y_1} = \frac{\mu_1 \sin\theta_1}{\mu_2 \sin\theta_2} \qquad \text{..........35}$$

Elimination of coma is possible if the lateral magnification $\frac{y_2}{y_1}$ is the same for all rays of light, irrespective of the shape angles θ_1 and θ_2. Thus coma can be eliminated if, $\frac{\sin\theta_1}{\sin\theta_2}$ is a constant because $\frac{\mu_1}{\mu_2}$ is constant. A lens that satisfies the above condition is called an aplanatic lens.

Astigmatism:

Astigmatism, similar to coma, is the aberration in the image formed by a lens, of object points off the axis. The difference between Astigmatism and coma, however, is that in coma the spreading of the image takes place in a plane perpendicular to the lens axis and in Astigmatism the spreading takes place along the lens axis. **Astigmatism** discussed in this article is different from the one treated in defective vision.

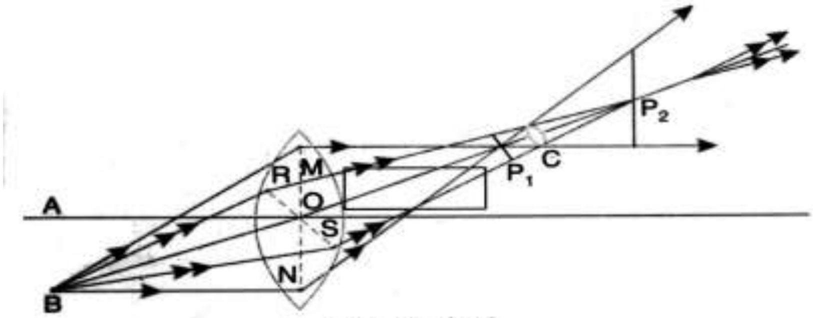


Figure : (21)

Figure (21), illustrates the defect of Astigmatism in the image of a point B situated off the axis. Two portion of the cone of rays of light diverging from the point Bare taken.

The cone of the rays of light refracted through the tangential (vertical) plane BMN comes to focus at point P_1 nearer the lens and the cone of rays refracted through the sagittal (horizontal) plane BRS comes to focus at the point P_2 away from the lens. All rays pass through a horizontal line passing through P_1 called primary image and also through a vertical line passing through P_2 called secondary image. The refracted beam has an elliptical cross-section , which ends to a horizontal line at P_1 and a vertical line at P_2.the cross section o the refracted beam is circular at some point between the primary and secondary images and this is called the **circle of least confusion**. If a screen is held perpendicular to the refracted beam between the points P_1 and P_2 the shape of the image at different positions is as shown in the figure (22) here.

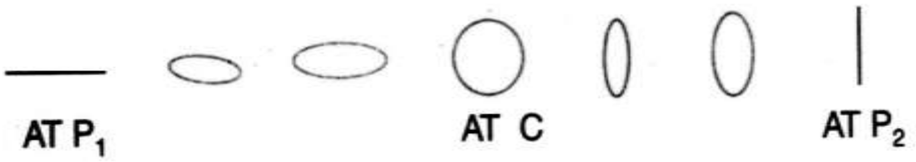


Figure : (22)

The locus o the primary image of all points in the object plane gives the surface of revolution about the lens axis and is called the primary image surface. The locus of the secondary images gives the secondary image surface. The surface of best focus is given by the locus of the circles or least confusion.

The primary and the secondary image surfaces and the surface of the best focus are illustrated in the figure (23).

P_1 and P_2 are the images of object point B. TPN and SPR are the first and the second image surfaces and KPL is the surface of the best focus. The three surfaces touch at the point P on the axis. Generally the surface of the best focus is not plane but curved as shown. This defect is called the curvature of the field. The shape of the image surfaces depends on the shape of the lens and the position of the stops.

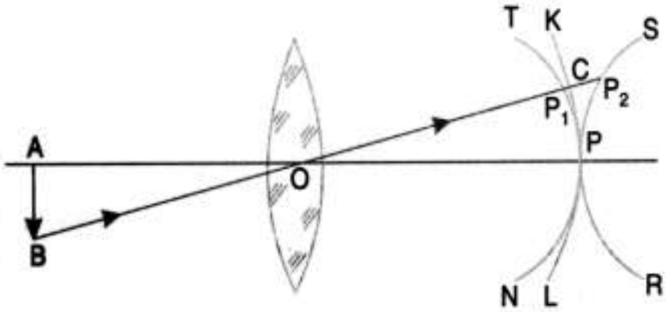


Figure : (23)

If the primary image surface is to the left of the secondary image surface astigmatism is said to be positive, otherwise negative.

By using a convex and a concave lens of suitable focal length and separated by a distance, it is possible to minimize the astigmatic difference and such a lens combination is called an anastigmat.

Curvature of the field:

The image of an extended object due to a single lens is not a flat one but it will be a curved surface. **The central portion of the image nearer the axis is in focus but the outer regions of the image away from the axis are blurred. This defect is called the curvature of the field. This defect is due to the fact that the paraxial focal length is greater than the marginal focal length.**

This aberration is present even if the aperture of the lens is reduced by a suitable stop, usually employed to reduce spherical aberration, coma and astigmatism. The figure illustrates the presence of curvature of the field in the image formed by a convex lens. A real image formed by a convex lens curves towards the lens (see figure, 24) and a virtual image curves away from the lens (see figure, 25).

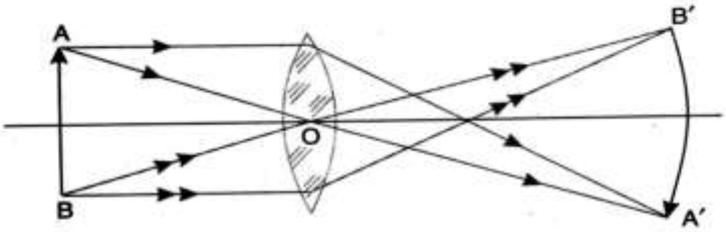


Figure : (24)

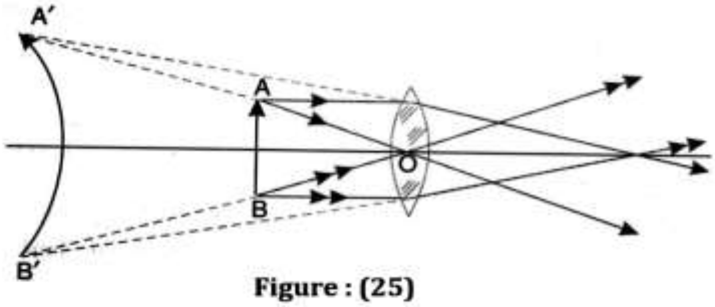


Figure : (25)

Figure (26) represents the curvature of the field present in the image formed by a curvature of the field present in the image formed by a concave lens.

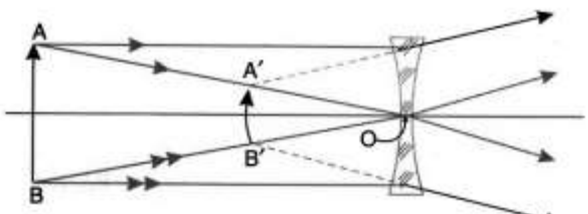


Figure : (26)

For a system of thin lenses, the curvature of the final image can, theoretically, be given by the expression

$$\frac{1}{R} = \sum \frac{1}{\mu_n f_n} \qquad \text{.......36}$$

Where R is the radius of the final image, μ_n and f_n are the refractive index and focal length of the n^{th} lens. For the image to be flat, R must be infinity

$$\therefore \quad \frac{1}{R} = \sum \frac{1}{\mu_n f_n} = \frac{1}{\infty} = 0$$

Correspondingly, the condition for two lenses placed in air, reduces to

$$\frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} = 0 \qquad \text{.....37}$$

This is known as Petzwal's condition for no curvature. This condition holds well whether the lenses are separated by a distance or placed in contact. As the refractive indices μ_1 and μ_2 are positive, the above condition will be satisfied if the lenses are of opposite sign. If one of the lenses is convex the other must be concave.

Astigmatism and coma are completely eliminated if the primary and secondary image surfaces are coincident the plane. In this case, the surface of the best focus will also be a plane one. But this cannot be achieved with a single lens. Astigmatism or curvature of the field can be minimized by introducing suitable stops on the lens axis. If the primary and the secondary image surfaces are made to have equal and opposite curvatures as shown in figure (27), the surface o the best focus will be plane and midway between them.

Astigmatism will, however, be present. Astigmatism can be eliminated by having the same curvature for the primary and secondary image surfaces. In this case curvature of the field will be present see figure (28).

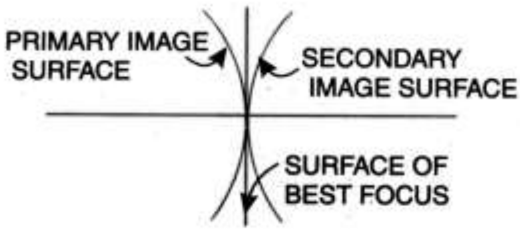


Figure : (27)

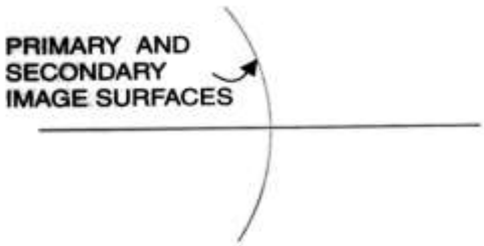


Figure : (28)

Correction for coma is more important than astigmatism for object points having comparatively small angular distance from the axis. Hence, telescope objectives, whose field of view small, are corrected for coma rather than for astigmatism. On the other hand camera lens of wide field has to be necessarily corrected for astigmatism.

Distortion:

The failure of a lens to form a point image due to a point object is due to the presence of spherical aberration, coma and astigmatism. The variation in the magnification produced by a lens for different axial distances results in an aberration is called **distortion.**

The aberration is not due to lack of sharpness in the '**image'.** Distortion is of two types a) pi-cushion distortion and b) barrel-shaped distortion. In pi-cushion distortion, the magnification increases with increasing axial distance and the image of an object (see figure 29a) appears as shown in figure (29 b). On the other hand, if the magnification decreases with increasing the axial distance, it results in barrel-shaped distortion and the image appears as shown in figure (23 c).

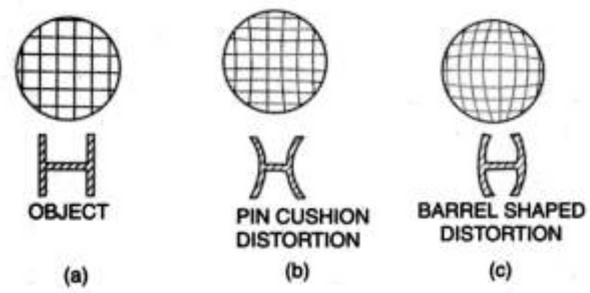


Figure : (29)

In the case optical instruments intended mainly for visual observation , a little amount of distortion may be present but it must be completely eliminated in a photographic camera lens, where the magnification of the various regions of the object must be the same.

In the absence of stops, which limit the cone of rays or light striking the lens , a single lens is free from distortion. But, if stops are used , the resulting image is distorted.

If a stop is placed before the lens the distortion is barrel-shaped see figure (30) and if stop is placed after the lens, the distortion is pin-cushion type see figure (31). To eliminate distortion a stop is placed between two symmetrical lenses, so that the pin-cushion distortion produced by the first lens is compensated by the barrel-shaped distortion produced by the second lens see figure (32). Projection and camera lenses are constructed in this way.

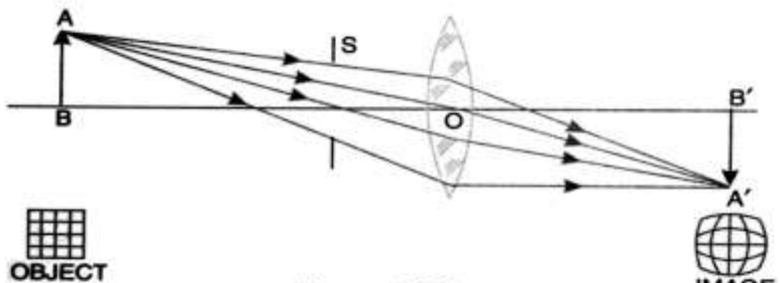


Figure : (30)

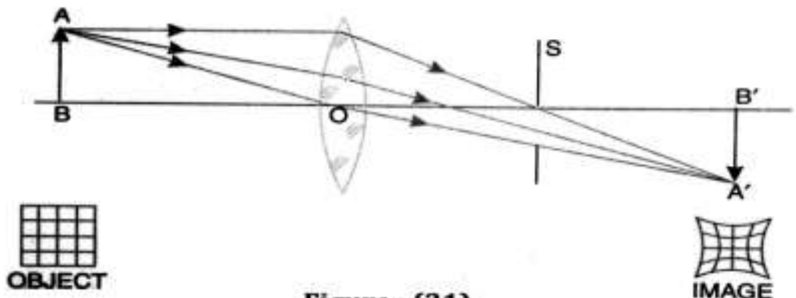


Figure : (31)

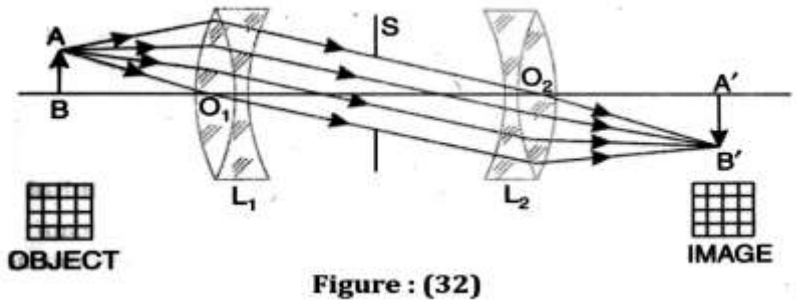


Figure : (32)

Chromatic aberration in a lens:

A single lens produces a coloured image of object illuminated by white light and this defect is called chromatic aberration. Elimination of this defect in a system of lenses is called achromatism.

(a) Expression for longitudinal chromatic aberration for an object at infinity:

When a parallel beam of white light is passed through a lens, the beam gets dispersed and rays of light of different colours (wavelength) come to focus at different points along the axis. The violet rays of light come to focus at a point nearer the lens and the red rays of light at a farther point see figure (33). f_v is the focus for the violet rays and f_r is the focus for the red rays. The colours in between violet and red come to focus between f_v and f_r. The distance (f_r - f_v) =x is called the **longitudinal or axial chromatic aberration**.

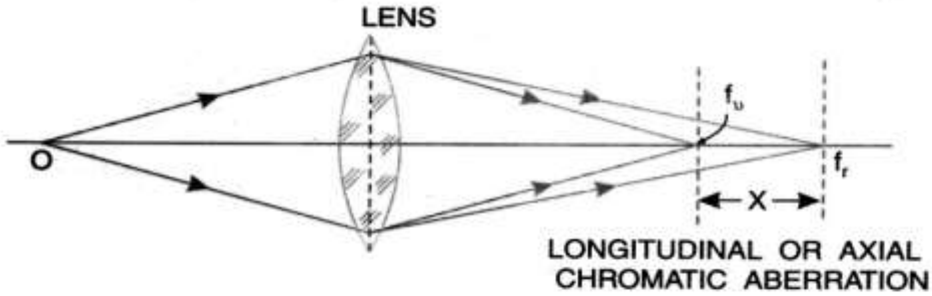


Figure : (33)

The focal length of a lens is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{……….38}$$

or $$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f(\mu - 1)} \qquad \text{……….39}$$

Similarly

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{……….40}$$

$$\frac{1}{f_v} = \frac{(\mu_v - 1)}{(\mu - 1) f} \qquad \text{………..41}$$

$$\frac{1}{f_r} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{………..42}$$

$$\frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu - 1) f} \qquad \text{………..43}$$

From equation (40) and (42), we get

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{1}{(\mu - 1) f}(\mu_v - 1 - \mu_r + 1)$$

$$\frac{f_r - f_v}{f_v f_r} = \frac{(\mu_v - \mu_r)}{(\mu \; -1) f}$$

Considering $f_v f_r = f^2$, where f is the mean focal length and no one can write

$$\frac{f_r - f_v}{f^2} = \frac{(\mu_v - \mu_r)}{(\mu - 1) f}$$

$$f_r - f_v = \frac{(\mu_v - \mu_r) f}{(\mu - 1)}$$

$$f_r - f_v = \omega f \qquad \text{……….44}$$

Where $\omega = \dfrac{(\mu_v - \mu_r)}{(\mu \; -1)}$ is known as the dispersive power of the material.

Thus the axial chromatic aberration for a thin lens for an object at infinity is equal to the product of the dispersive power of the material of the lens and f is the mean focal length. It is also clear, here that a single lens cannot form an image free from chromatic aberration.

(b) Expression for the longitudinal chromatic aberration for an object at finite distance:

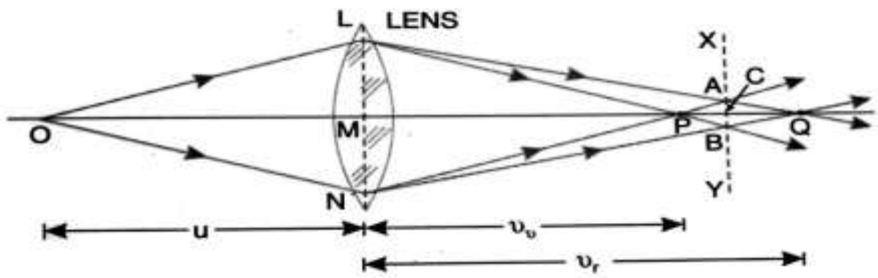


Figure : (34)

Consider a point object illuminated by white light and situated on the axis of the lens. Coloured images are formed along the axis. The violet image is nearest the lens and the red image is the farthest. In between these two images as shown in given figure (34), if a screen is placed at the position XY, the image of least chromatic aberration is formed.

Let u be the distance of the object and v_v and v_r the distance of the violet and red images on the axis of the lens. If fv and f_r are the focal lengths for the violet and red rays if the light then

$$\frac{1}{v_v} - \frac{1}{u} = \frac{1}{f_v} \qquad \text{……….45}$$

$$\frac{1}{v_r} - \frac{1}{u} = \frac{1}{f_r} \qquad \text{……….46}$$

Subtracting eq. (46) from (45)

$$\frac{1}{v_v} - \frac{1}{v_r} = \frac{1}{f_v} - \frac{1}{f_r}$$

$$\frac{v_r - v_v}{v_v v_r} = \frac{f_r - f_v}{f_r f_v}$$

Taking $v_v v_r = v^2$, and $f_v \, f_r = f^2$

$$\frac{v_r - v_v}{v^2} = \frac{f_r - f_v}{f^2}$$

But$f_r \ - \ f_v \ = \omega f$,

$$\frac{v_r - v_v}{v^2} = \frac{\omega f}{f^2} = \frac{\omega}{f}$$

So $$v_r - v_v \quad = \quad \frac{\omega v^2}{f} \qquad \text{……….47}$$

It is clearly seen in this case that the longitudinal chromatic aberration depends on the distance of the image and hence on the distance of the object from the lens, in addition to its dependence on the dispersive power and the focal length of the lens.

Objectives and Eyepieces:

Importance of an objective lens

We have earlier studied about the simple microscope. We have studied that the magnification power of a simple microscope can be increased by decreasing the focal length of the lens. However, the focal length of a lens cannot be decreased beyond a certain limit. Moreover, the lens of a small focal length has a smaller diameter because the curvature of the surface is large and the field of view is small.

Therefore, to increase the magnifying power, two separate lenses are used. The lens near the object is called **objective**, which forms a real image of an object under examination see figure (35).

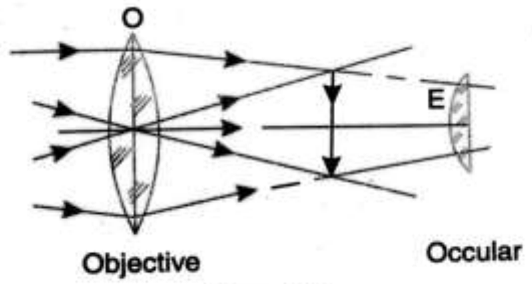


Figure : (35)

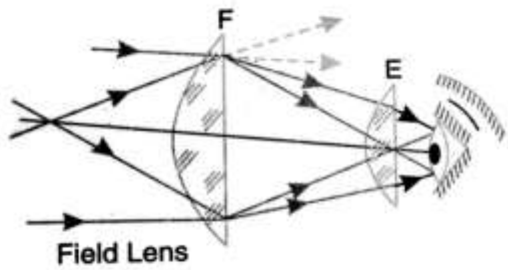


Figure : (36)

The lens used to enlarge this image further to form a final image and which is then viewed by the eye, is called **eyepiece or ocular.** The objective of a ordinary microscope is usually an achromat with a magnification of about × 5 and an eyepiece usually consists of multiple lenses.

An optical instrument is required to produce a magnified image free from aberration and a bright image covering a wide field of view. If a single lens is used as an eyepiece, the final image will suffer from spherical and chromatic aberrations.

Another drawback is the small field of view, which becomes lesser and lesser as the magnification of the instrument is increased, The rays passing through the outer portions of the image are refracted through the peripheral portion s of the eye lens and they cannot simultaneously enter the small aperture of the pupil of the eye placed closed to the eye lens see figure (35).

Hence only that part of the image which is nearer to the axis will be seen. Therefore, the final image will cover a small field of view. The field of view will progressively decrease as the distance between the objective and ocular is increased. The distance is varied in order to increase the magnification. In other words, the greater the magnifying power of the lens in the eyepiece to cause all the rays from the image to enter the eye lens. This extra lens is called a **field lens.**

The function of the field lens is to gather in more of the rays from the objective toward the axis of the eye piece see figure (36).

The field lens and the eye lens together constitute an ocular or eyepiece. Two lenes are made and kept in such a way that their combination is achromatic and free from spherical aberrations. Two common eyepieces are the Huygens and Ramsden types.

Huygens eyepiece:

In the Huygens eyepiece a converging beam enters the field lens and forms a virtual image before the eye lens. The need for a converging beam implies that this eyepiece does not act like a simple magnifier.

- **Construction**

Huygens eyepiece consists of two lenses having focal lengths in the ratio 3:1 and the distance between them is equal to the difference in their focal lengths. The focal lengths and the positions of the two lenses as shown in figure (37) are such that each lens produces an equal deviation of the ray and the system is achromatic. This eyepiece is free from chromatic as well as spherical aberration as it satisfies the two conditions simultaneously.

i) The lens combination acts as achromatic system if $D = \frac{f_1 + f_2}{2}$, where D is the distance between the lenses and f_1 and f_2 are the focal length of the two lenses.

ii) The lenses produces equal deviations of the incident ray when the distance between the two lenses is equal to $(f_1 - f_2)$.

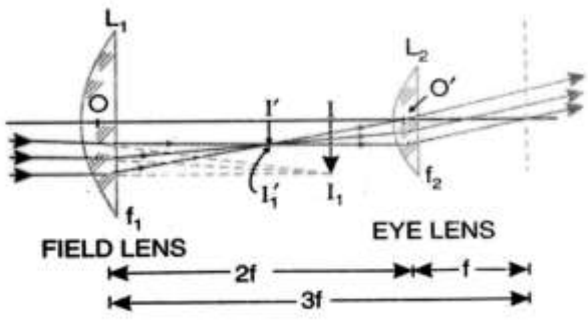


Figure : (37)

The field and the eye lenses used are Plano convex and are placed with their convex surfaces towards the incident ray. In this way, the total deviation due to the combination is divided into four parts which makes the combination to have minimum spherical aberration.

In spherical and chromatic aberrations are to be minimized simultaneously, the following condition is to be satisfied.

$$D = \frac{f_1 + f_2}{2} \qquad \text{Chromatic aberration}$$

$$D = f_1 - f_2 \qquad \text{Spherical aberration}$$

Combining the two aberrations, we obtain

$$\frac{f_1 + f_2}{2} = f_1 - f_2$$

$$f_2 \quad = 3 f_1$$

or $\therefore D = 3f_1 - f_1 = 2f_1$ ……….48

Thus satisfy the conditions for minimum chromatic and spherical aberrations, the focal length of the field lens should be three times the focal length of the eye lens and the distance between them should be equal to twice the focal length of the eye lens. Huygens' eyepiece is constructed on this principle.

Theory

The objective forms an image, which serves as a virtual object for the field lens. The field lens forms real inverse image I'I'$_1$. If this image is situated at the principal focus of the eye lens, then the final image is at infinity.

Equivalent focal length:

The equivalent focal length of the eyepiece can be found as follows. If F is the equivalent focal length of the eyepiece, then it is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{3f} - \frac{2f}{3f^2} = \frac{2}{3f}$$

$$F = \frac{3f}{2}$$

………..49

The equivalent lens leis behind field lens at a distance of

$$x = \frac{d \times F}{f} = \frac{2f \times \frac{3f}{2}}{f} = 3f \qquad \text{………..50}$$

In other words, the equivalent lens is at a distance of $3f - 2f = f$ behind the eye lens.

Position of cross wire:

It has been observed that the principal focus of the equivalent lens lays ƒ/2 ahead of the eye lens. It is here that the image due to objective must be formed in order that the final image is at infinity. The rays coming from the objective are, however, intercepted by the field lens, thus displacing the image to position I$_1$. It is here that the cross-wire should be placed. We further noted above that the image I I$_1$ is at a distance of ƒ/2 from the eye lens or 3 ƒ/2 from the field lens. Hence, for field lens u = -3 ƒ/2. Then

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore -\frac{2}{3f} + \frac{1}{v} = \frac{1}{3f}$$

or $\quad v = f$ …………51

In other words, II$_1$ lies midway between the two lenses and so fixes the position of the cross wire or scale, if used. Since the image found by the objective lies behind the field lens (instead of in the front) this eyepiece is sometimes referred to as a negative eyepiece. It may be noted that the image of the scale is formed by the eye lens where as the final image is produced by both the lenses. Hence the image and the scale would not be magnified equally and so the measurements will not be reliable. The image of the cross wire or scale would have all the defects of an image formed by a single lens. Hence in instruments using Huygens' eyepieces, scale is not used except when the magnification is low.

Merits and Demerits

i) Huygens' eyepiece is fully free from chromatic aberration because the distance between the lenses is equal to half the sum of their focal lengths.
ii) Minimum Spherical aberration, because the distance between the two lenses is equal to the difference of their focal lengths.

iii) The field of view of this eyepiece is smaller than that of Ramsden's eyepiece.

Ramsden eyepiece:

Ramsden's eyepiece consists of two Plano convex lenses each of focal length ƒ separated by a distance equal to (2/3) ƒ . The lenses are kept with their curved surface facing each other, as shown in the figure (38), thereby reducing spherical aberration. The field lens is a little larger than the intermediate image and is placed close to this image to as much light as possible to pass through it. The eye lens has a smaller diameter but carries out the actual magnification.

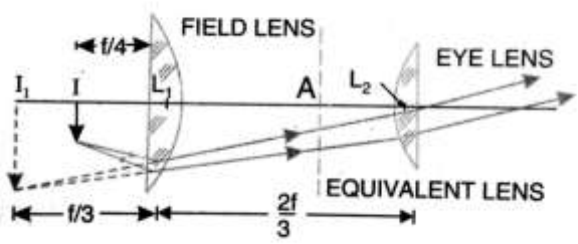


Figure : (38)

Theory

The objective forms the real inverted image I of a distant object. This serves as an object for the field lens, which gives rise to a virtual image I_1. I_1 in turns serves as an object for the eye lens, which gives the final image at infinity, because I_1 le made to lie at its principal focus.

Equivalent focal length:

The equivalent focal length of the eyepiece can be found as follows. If F denotes the focal length of the equivalent lens, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{\frac{2}{3}f}{f^2} = \frac{2}{3f}$$

$$= \frac{2}{f} - \frac{2}{3f} = \frac{4}{3f}$$

$$\therefore \quad F = \frac{3f}{4} \qquad \text{........52}$$

The equivalent lens of focal length $F = \frac{3f}{4}$ must be placed behind the field lens at a distance

$$\alpha = \frac{F \times d}{f} = \frac{(\frac{3}{4})f \times (\frac{2}{3})f}{f} = \frac{f}{2} \qquad \text{......53}$$

Thus the equivalent lens lies between the field lens and the eye lens.

Position of the cross wire:

The cross wire should be placed at the position of I. Now, the position of I relative to field lens can be found as follows.

If the final image is to be formed at infinity, the image I should lie in the focal plane of the equivalent lens. In other words, the distance AI should be equal to F= (3 ƒ) / 4. Since $AL_1 = ƒ/2$, $I_4 = ƒ/4$. Therefore, the objective should produce the image at a distance of ƒ/4 in front of the field lens. A fine scale may be placed here if is desire to measure the size of the image. Since the scale and image would be magnified equally, the measurement would be trustworthy.

Merits and Demerits:

i) The field of view of this eyepiece is fairly wide.

ii) It is not entirely free from chromatic aberration since the distance between the two lenses is not equal to half the sum of their focal lengths. However, chromatic aberration is minimized by using an achromatic combination both for the field lens and the eye lens.

iii) Spherical aberration is minimized by using two Plano-convex lenses thereby spreading deviation over four surfaces.

Ramsden's eyepiece is used practically in all instruments where measurements of the size of the final image are to be made. The eyepiece is sometimes referred to as positive eyepiece because a real image is formed by the objective in front of the field lens, which further acts as a real object for the eye lens.

Comparisons of Ramsden eyepiece and Huygens eyepiece:

	Ramsden eyepiece	**Huygens eyepiece**
1	Ramsden eyepiece is positive eyepiece. The image formed by the objective lies in front of the field lens. Therefore, cross-wire can be used.	Huygens eyepiece is negative eyepiece. The image formed by the objective lies in between the two lenses. Therefore, cross-wire cannot be used.
2	The condition for minimum spherical aberration is not satisfied. But by spreading the deviations over four surfaces, spherical aberration is minimized.	The condition for minimum spherical aberration is satisfied.
3	It does not satisfy the condition for achromatism but can be made achromatic by using an achromatic doublet as the eye lens.	It satisfies the condition for achromatism.
4	It is achromatic for only two chosen colours.	It is achromatic for all colours.
5	The other types of aberrations are better eliminated. Coma is absent and distortion is 5% higher.	The other types of aberrations like pincushion distortion are not eliminated.
6	The eye clearance is 5% higher,	The eye clearance is too small and less comfortable.